

Tropical wave modes modified by semi-empirical parameterizations of moisture



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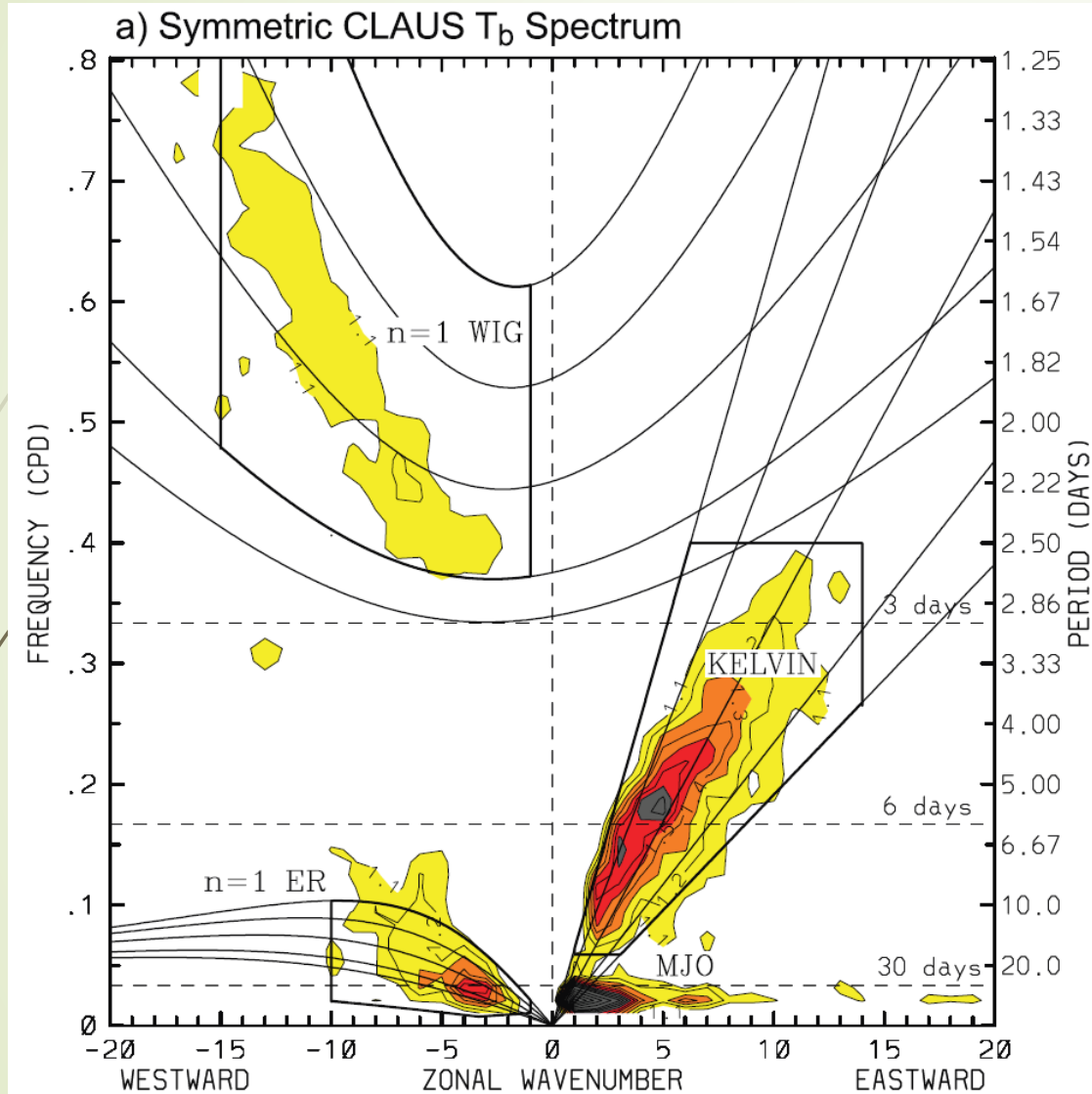
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“Separation” between
MJO and classical
waves

Kiladis et al. 2009

Modified dynamic modes

- Moisture processes are considered in the dynamic equations
 - convergence in the planetary boundary layer; surface heat flux;
 - convectively coupled Kelvin-Rossby wave; multi-scale interaction;
 - ocean-atmosphere interaction; ...
- Take Kang et al. (2013) as an example

$$\frac{\partial u}{\partial t} - \beta y v + \frac{\partial \phi}{\partial x} = -\kappa \nabla^2 u; \quad \beta y u + \frac{\partial \phi}{\partial y} = 0$$

$$\frac{\partial \phi}{\partial t} + c_0^2 \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = \mathcal{Q} - N\phi$$



The heating term
Related to moisture processes

Moisture mode

- ▶ The modes determined by the moisture tendency
- ▶ Take Sobel and Maloney (2012) for an example

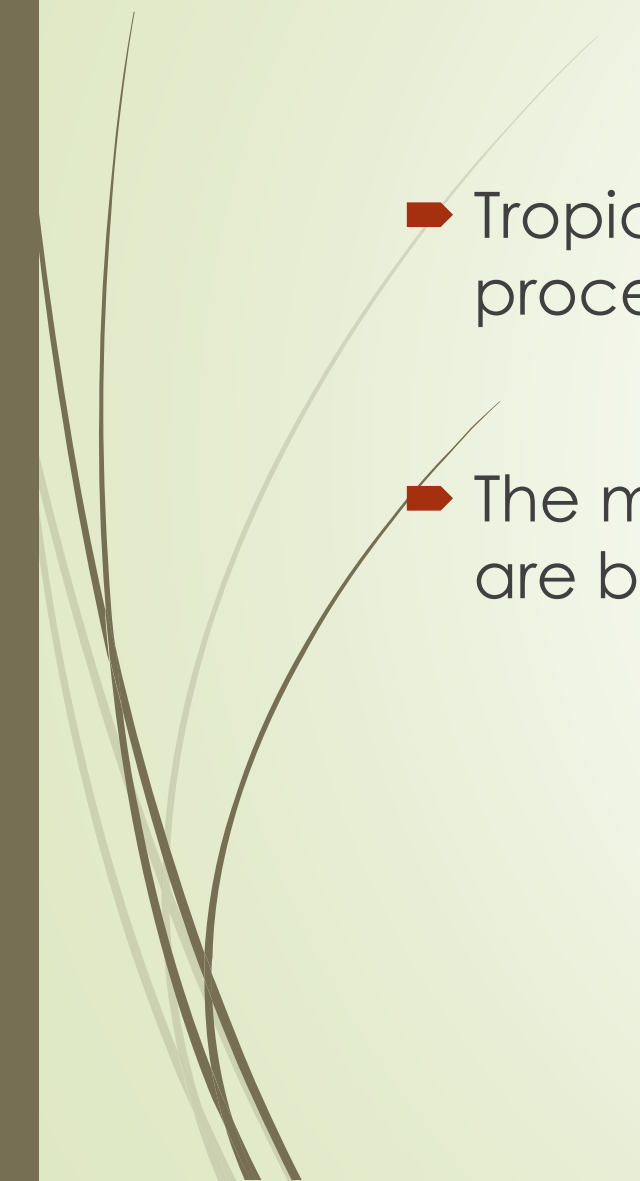
Time tendency of moisture

$$\frac{dW}{dt} = \frac{\partial W}{\partial t} + u \frac{\partial W}{\partial x} = -\tilde{M}P + E - (1 - \tilde{M})R + k_w \frac{\partial^2 W}{\partial x^2}$$

$$\hat{u}(x, t) = \int G(x|x') [P(x', t) - R(x', t)] dx'$$



Motivation

- Tropical wave modes under the influence of dynamical processes and the moisture processes;
 - The momentum tendency and the moisture tendency are both considered in a uniform theoretical framework;
- 

Shallow-water system

$$\frac{\partial \vec{u}}{\partial t} + \beta y \vec{k} \times \vec{u} = -\nabla \phi,$$

$$\frac{\partial \phi}{\partial t} + c^2 \nabla \cdot \vec{u} = Q,$$

$$\frac{\partial W}{\partial t} + u \frac{\partial W}{\partial x} + v \frac{\partial W}{\partial y} - M \nabla \cdot \vec{u} = m_b + E - P,$$

\vec{u} is the horizontal velocity,
 c is the gravity wave speed,

E denotes evaporation,

$$W = \int \frac{q}{g} dp,$$

m_b is the moisture source due to Ekman pumping.

ϕ is the geopotential,
 M is the moisture stability.

P denotes precipitation

$$Q = Q(P).$$

Parameterization for evaporation

The traditional bulk formula

$$E = \rho_0 C_d |\bar{U} + u| (q_s - q_a)$$

Assuming a dominant mean westerly wind (positive \bar{U}) in the background, for ex., from the tropical Indian Ocean to the western Pacific Ocean, \bar{U} is positive and overwhelms u . Thus, $|\bar{U} + u| = \bar{U} + u$, regardless of the direction of u .

$$E = bu,$$

$$b = \rho_0 C_d (q_s - q_a)$$

Parameterization for precipitation

- ▶ convective precipitation due to the moisture transfer from the PBL

$$-\mu \cdot \nabla \vec{u}_b \cdot \bar{W}$$

- ▶ large-scale precipitation proportional to the water vapor content in the air column

$$d_3 W$$

- ▶ precipitation due to convectively available potential energy

$$d_4 \phi$$

Parameterization for precipitation

$$P = -\mu \nabla \vec{u}_b \cdot \bar{W} + d_3 W + d_4 \phi,$$

According to Wang and Li (1994)

$$\nabla \cdot \vec{u}_b = -\frac{(\nabla^2 \phi + \beta u + \beta \delta v)}{\epsilon(1 + \delta^2)},$$

Finally, $P = -d_1 u - d_2 v + d_3 W + d_4 \phi$,

P is related to all state variables.

$$d_1 = \frac{(1-\mu)\bar{W}\beta}{\epsilon(1+\delta^2)} \text{ and } d_2 = \frac{(1-\mu)\bar{W}\beta\delta}{\epsilon(1+\delta^2)}.$$

Parameterized shallow-water system

$$\frac{\tilde{\partial}u}{\tilde{\partial}t} - \beta y v = -\frac{\partial\phi}{\partial x},$$

$$\frac{\tilde{\partial}v}{\tilde{\partial}t} + \beta y u = -\frac{\partial\phi}{\partial y},$$

$$\frac{\tilde{\partial}\phi}{\tilde{\partial}t} + c^2 \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) = -\delta_1 u - \delta_2 v + \delta_3 W + \delta_4 \phi,$$

$$\frac{\tilde{\partial}W}{\tilde{\partial}t} + u \frac{\partial \bar{W}}{\partial x} - M \nabla \cdot \vec{u}(x, y, t) = (b + d_1)u + d_2 v - d_3 W - d_4 \phi,$$

where $\delta_i = \frac{\mu}{\mu-1} \gamma L d_i$, $i = 1, 2$ and $\delta_i = \gamma L d_i$, $i = 3, 4$.

Eigen-value problem

$$(1 + i) \frac{\partial^2 V}{\partial y^2} + \left[\frac{\delta_1 y}{d_3 c} + i \left(\frac{\delta_3 b}{c^2 \beta} y + \frac{\delta_2}{c \sqrt{\beta c}} \right) \right] \frac{\partial V}{\partial y} - \left[(1 + i) y^2 - \frac{\delta_4}{d_3} y^2 + i \frac{\delta_2 y}{c \sqrt{\beta c}} \right] V = \Lambda V,$$

$$\Lambda = -(1 + i) \left(\omega^2 - k^2 + \frac{k}{\omega} \right) + \left[\gamma L \left(\frac{\delta_1 \omega}{\delta_3 c} - \frac{Mk}{c^2} \right) + i \frac{\delta_3 (b - \bar{W}_x)}{\omega \beta c^2} \right] \left(k - \frac{1}{\omega} \right) + \gamma L \frac{\delta_4 \omega^2}{\delta_3}.$$

$$\Lambda = \Lambda_d + \Lambda_m,$$

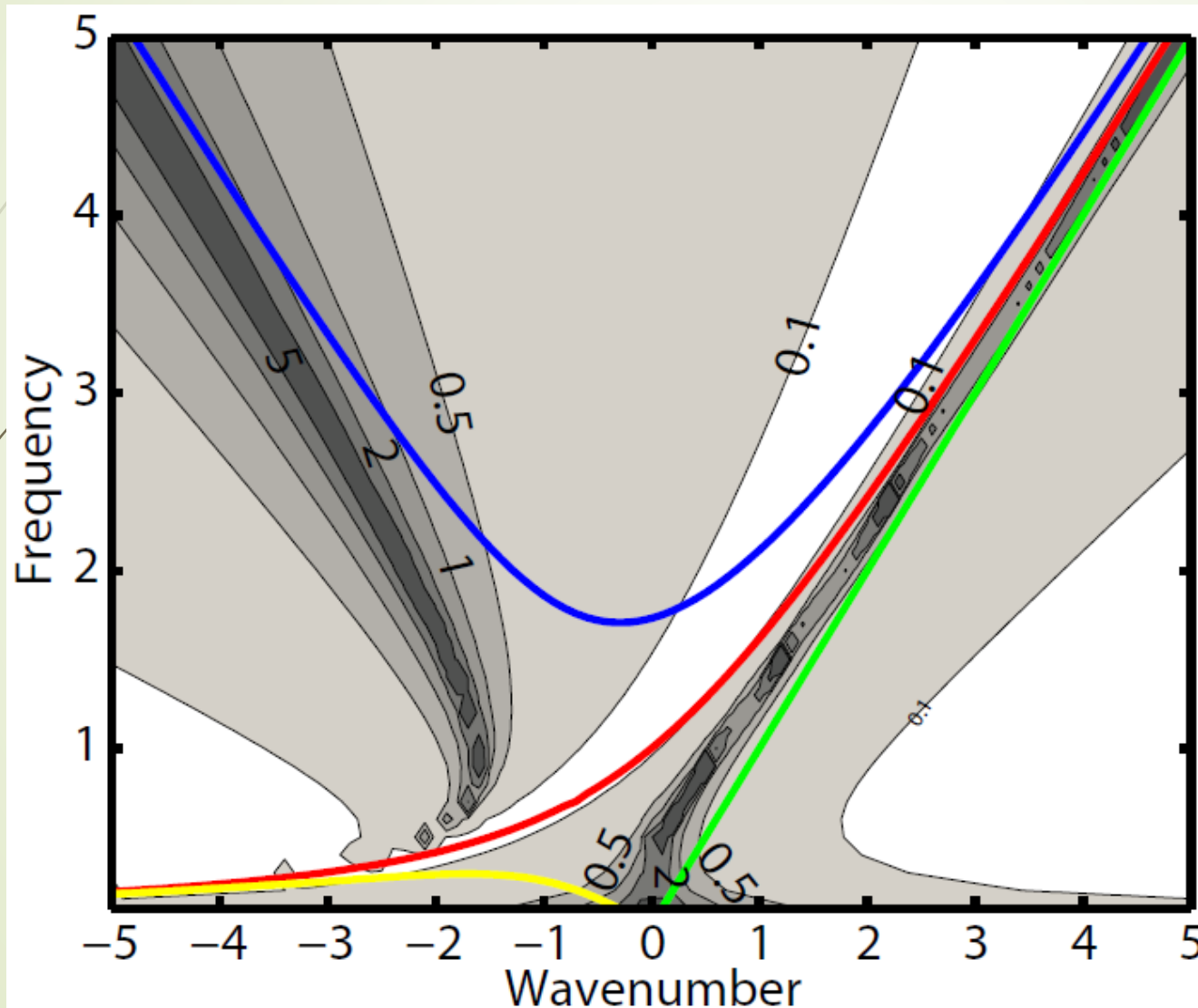
$$\Lambda_d = -(1 + i) \left(\omega^2 - k^2 + \frac{k}{\omega} \right),$$

$$\Lambda_m = \left[\gamma L \left(\frac{\delta_1 \omega}{\delta_3 c} - \frac{Mk}{c^2} \right) + i \frac{\delta_3 (b - \bar{W}_x)}{\omega \beta c^2} \right] \left(k - \frac{1}{\omega} \right) + \gamma L \frac{\delta_4 \omega^2}{\delta_3}.$$

Parameters

Parameter	Value	Parameter	Value
M	$-3 \times 10^{-5} \text{ kg m}^{-2}$	b	$2.3 \times 10^{-7} \text{ kg m}^{-3}$
c	50 m s^{-1}	\bar{W}_x	$-4.4 \times 10^{-8} \text{ kg m}^{-3}$
β	$2.3 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1}$	\bar{W}_y	$-1.7 \times 10^{-6} \text{ kg m}^{-3}$
d₁	$1.6 \times 10^{-5} \text{ kg m}^{-3}$	δ_1	$2.3 \times 10^{-3} \text{ m s}^{-2}$
d₂	$1.2 \times 10^{-5} \text{ kg m}^{-3}$	δ_2	$1.7 \times 10^{-3} \text{ m s}^{-2}$
d₃	$1.4 \times 10^{-4} \text{ s}^{-1}$	δ_3	$0.02 \text{ kg}^{-1} \text{ m}^4 \text{ s}^{-3}$
d₄	$2.7 \times 10^{-7} \text{ kg m}^{-4} \text{ s}$	δ_4	$3.9 \times 10^{-5} \text{ s}^{-1}$

Importance of moisture processes

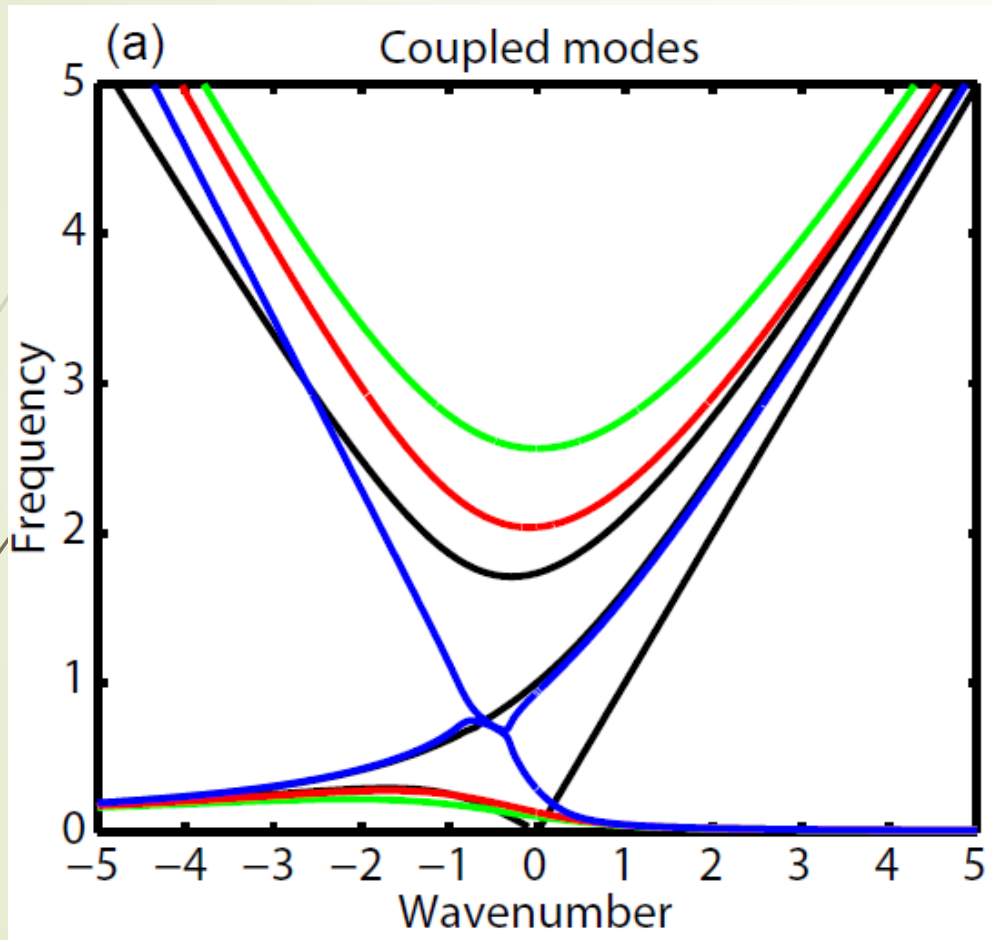


$$\Lambda = \Lambda_d + \Lambda_m,$$

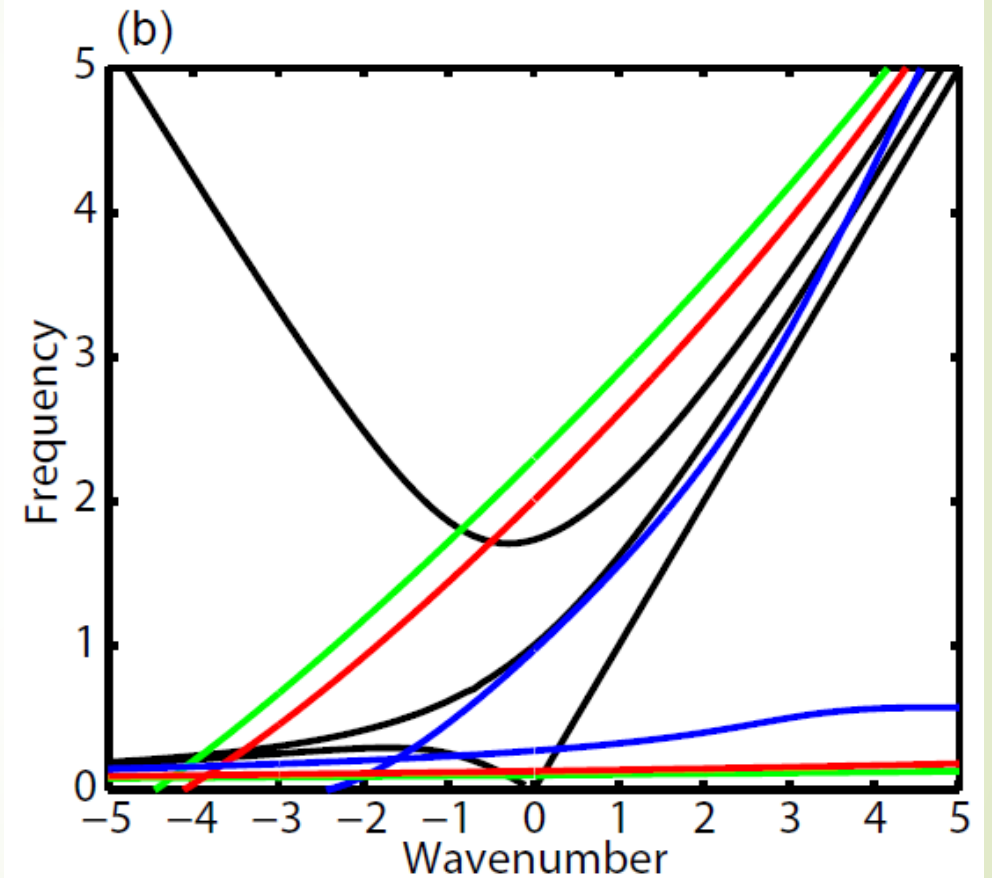
The ratio of
 $|\Lambda_m|/|\Lambda_d|$

When the ratio is
large, moisture
processes dominate

Dispersion relation

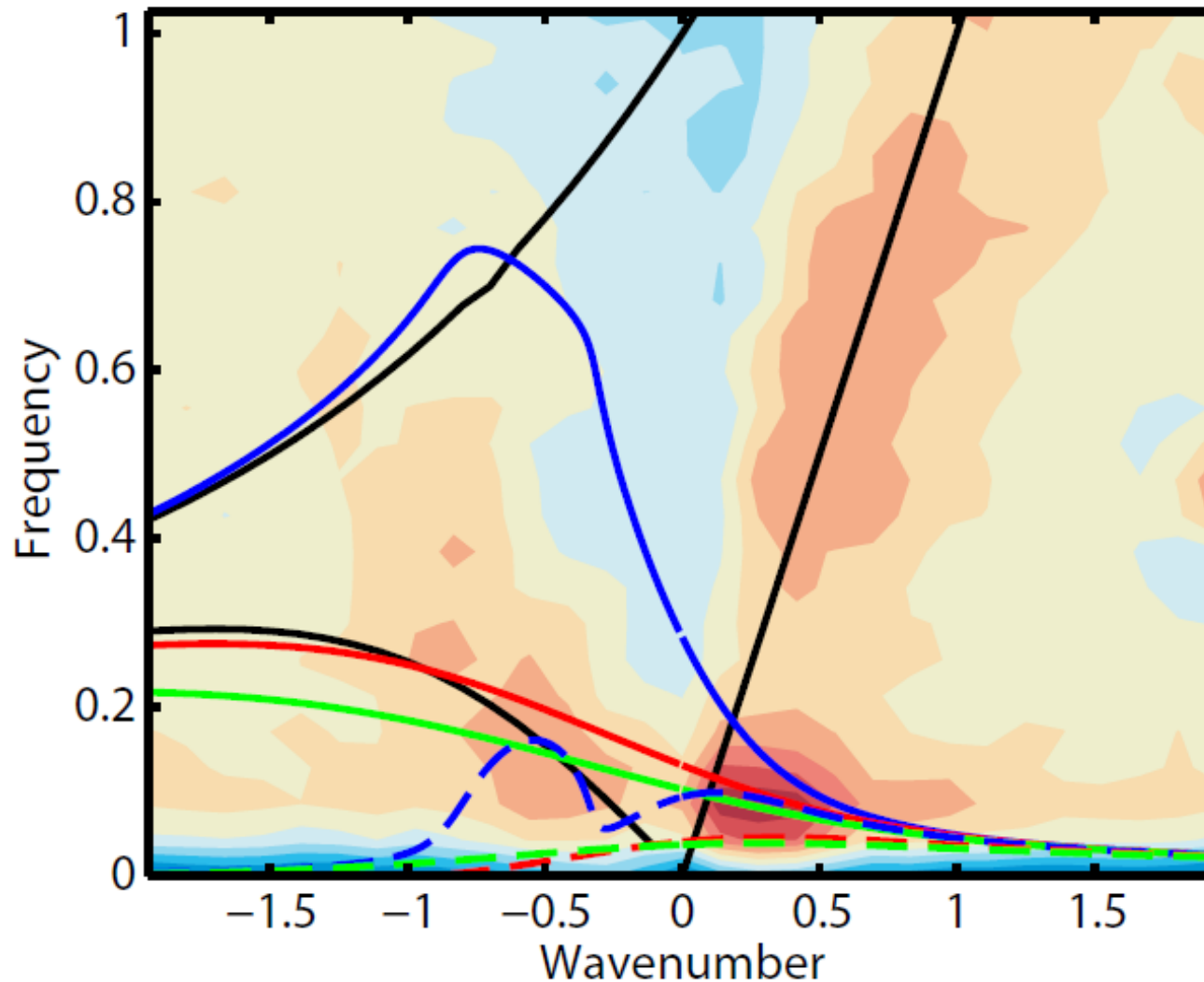


Black lines: classical dynamical waves;
Red lines: The second mode;



Blue lines: the first mode;
Green lines: the third mode.

The MJO domain



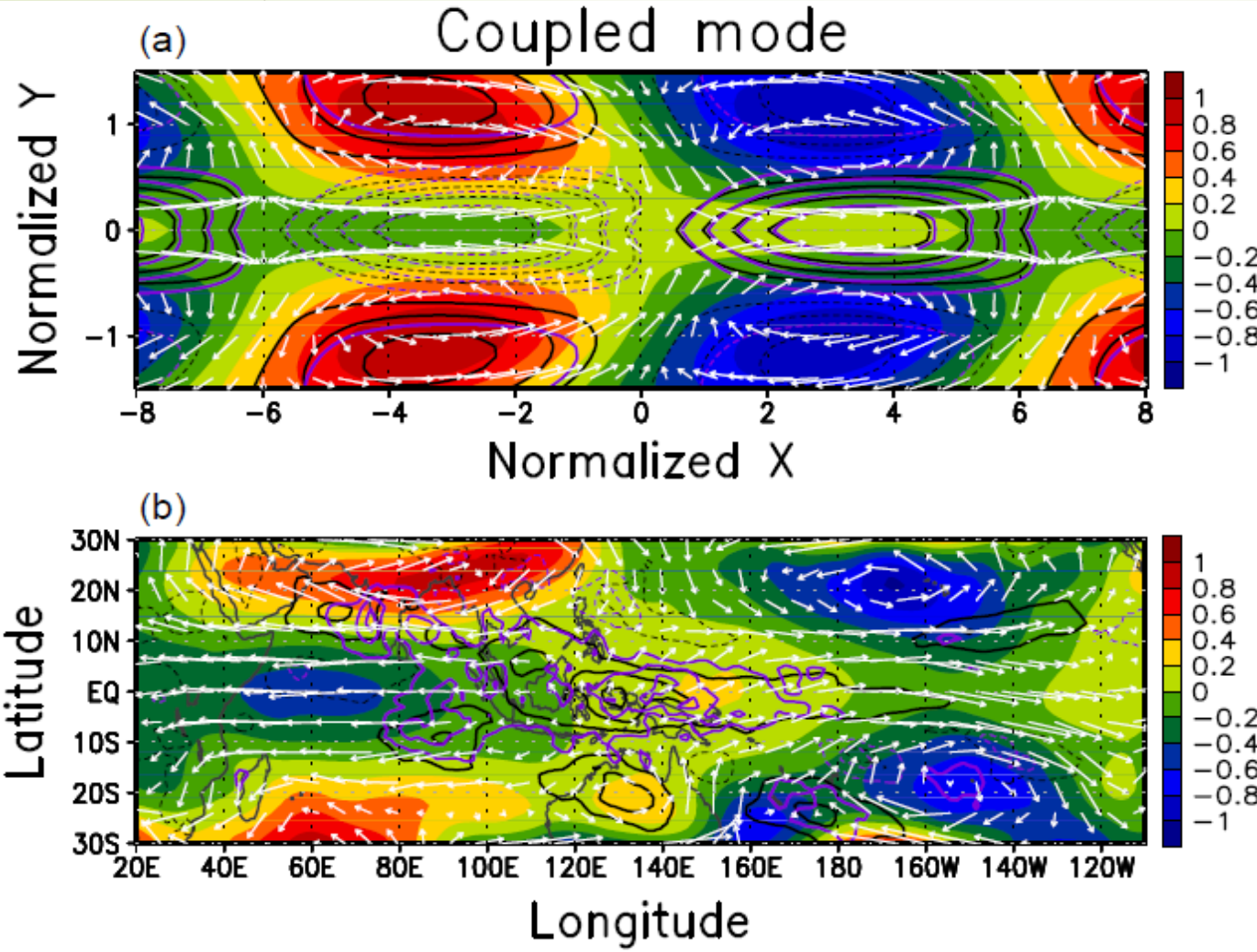
Black lines: classical dynamical waves

Blue lines: the first mode;

Red lines: The second mode;

Green lines: the third mode

Plan-view of the coupled mode



Colors $\sim \varphi$

Black contours $\sim W$

Purple contours \sim precipitation

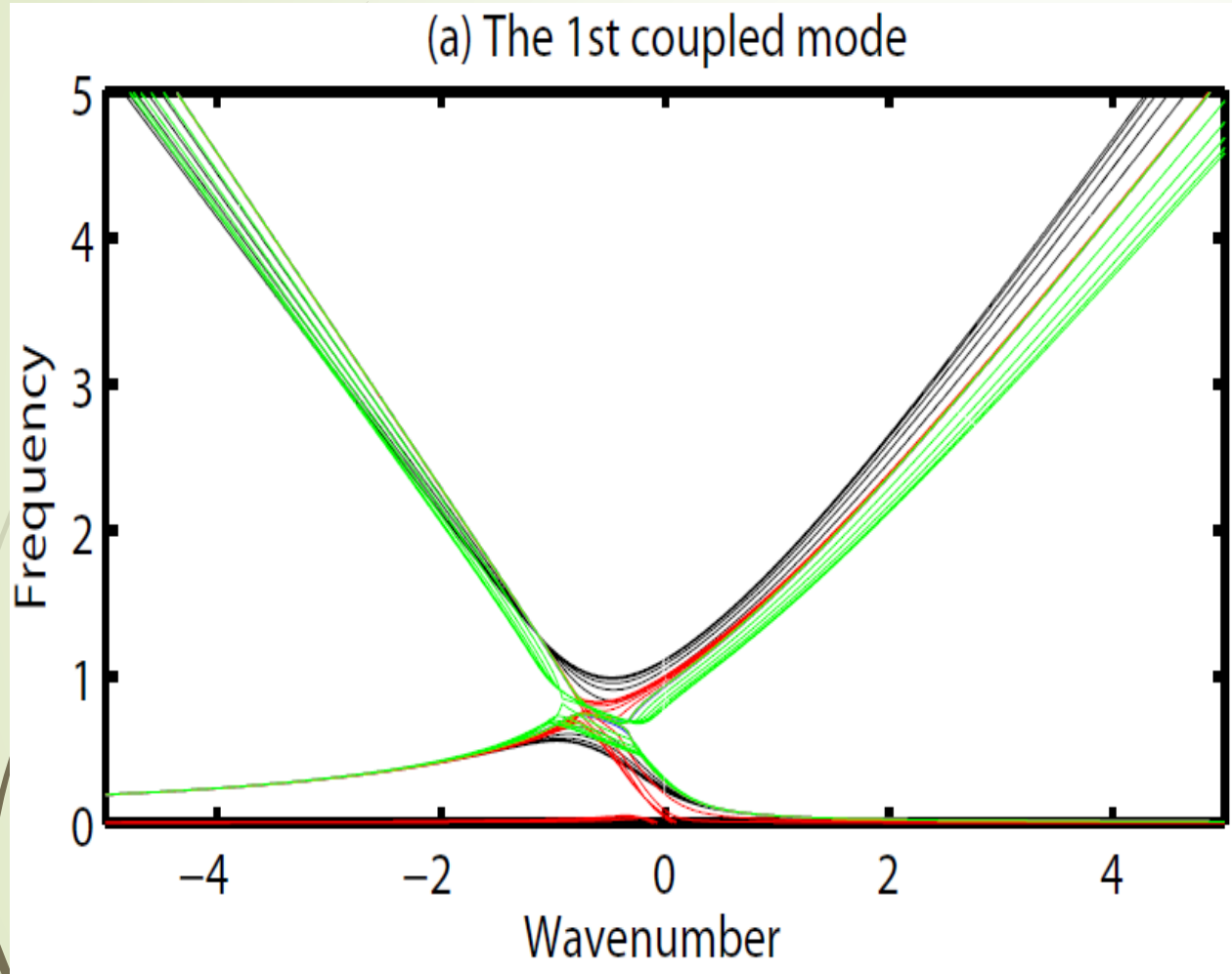
vectors $\sim \vec{u}$

Solid contours stand for positive anomalies

Dashed contours stand for negative anomalies

All variables are normalized with their own maximum

Parameter sensitivity



A parameter is perturbed by $\pm 50\%$ (from -50% to 50% with an interval of 10%) off their base values listed in the previous table.

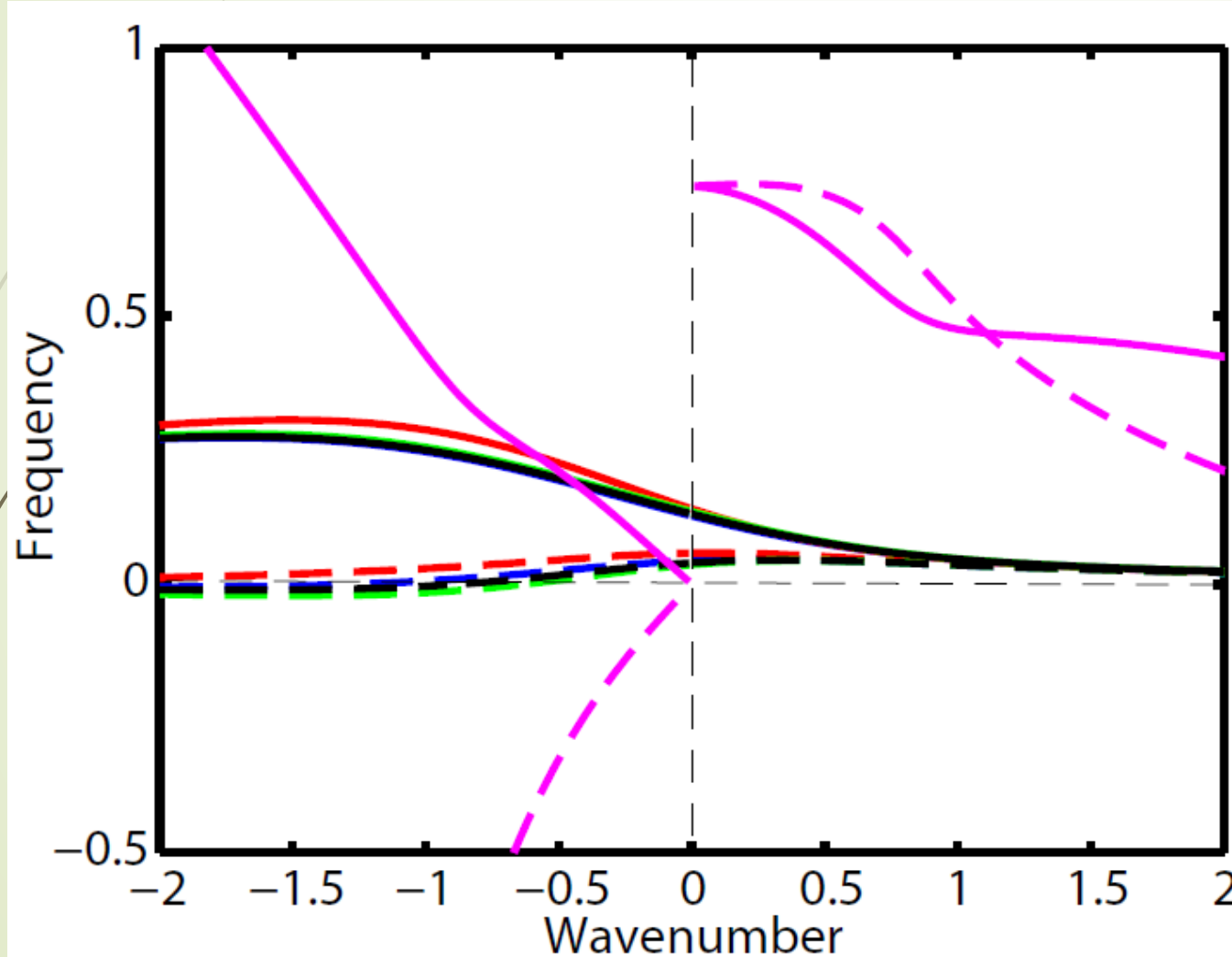
Black curves $\sim d_1, \delta_1$

Blue curves $\sim d_2, \delta_2$

Red curves $\sim d_3, \delta_3$

Green curves $\sim d_4, \delta_4$.

Influence of different parameter



Black lines show the dispersion relation with all parameters.

Blue lines: d_1 and δ_1 are zero

Red lines: d_2 and δ_2 are zero

Purple lines: d_3 and δ_3 are zero

Green lines: d_4 and δ_4 are zero



Conclusions

- ▶ With an ideal and linearized shallow water system, the effect of moisture on tropical wave modes is analytically studied;
- ▶ The moisture tendency and the momentum tendency are both explicitly considered;
- ▶ The coupled dynamic-moisture mode is consistent with the observed MJO in both the spectral and physical domains;
- ▶ New modes related to moisture processes occur when the moisture tendency is considered.



Thanks!